L1 Backup Navigation for Dual Frequency GPS Receiver

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Abstract

GPS satellites transmit a spread spectrum signal on L1 and L2 at 1575.42 and 1227.6 MHz, respectively. These different frequencies allow the user to estimate the ionosphere delay error, which is one of the main sources of navigation error. Dual frequency GPS receivers are therefore capable of generating refraction corrected (ionosphere free) measurements which can provide more accurate positioning than single frequency receivers. However, the GPS L2 signal is more difficult to acquire and track for two reasons: a) the broadcast signal strength is lower than on L1; and b) civilian receivers must use less efficient codeless or semicodeless techniques to recover the P2 code from the encrypted signal.

This paper presents a technique for L1 backup navigation in the event of intermittent loss of the L2 signal. A three-state Kalman filter is used for each GPS satellite. When both L1 and L2 signals are available for the satellite, L1 and L2 observables are used to estimate ionospheric refraction delay, delay rate, and a combination of integer ambiguities on L1 and L2 carrier-phase measurements. When the L2 signal is lost, the L1 signal is used together with the estimated states to synthesize L2 observables during the outage. When the L2 signal is re-acquired, state estimation is resumed. Special techniques are used to smoothly transition between the synthesized and real L2 observables. Experimental results are presented showing that dual frequency, refraction corrected navigation accuracy is sustained through significant periods of L2 signal loss.

The technique described herein could be applied to synthesize L1, L2 or L5 measurements if any one of the three frequencies is retained. The missing measurements at one of the frequencies can be synthesized using the measurements from the retained frequency that was not lost.

KEY WORDS: Kalman filter, ionosphere, cycle slip.

1 Introduction

A number of different techniques have been developed to obtain high-accuracy differential navigation using the GPS carrier-phase measurements. The highest accuracy technique is generally referred to as “real-time kinematic” (RTK) and has a typical accuracy of about one-centimeter. However, in order to obtain that accuracy, the whole-cycle ambiguity in the difference of the carrier-phase measurements must be determined. When the reference receiver is a substantial distance (more than a few tens of kilometers) from the navigation receiver it may become impossible to determine the whole-cycle ambiguity and the normal RTK accuracy cannot be achieved. Under these adverse circumstances the best that can be done is often to estimate the whole-cycle ambiguities as a real-valued (non-integer) variable. This practice is often referred to as determining a “floating ambiguity” value.

One method for determining the “floating ambiguity” value is to form the refraction corrected code and carrier-phase measurements, scale the carrier-phase measurements to the same units as the code measurements and subtract them from the code measurements. This “offset” value can be recursively averaged over time and becomes an increasingly accurate estimate of the “float-


2 Background

The section describes the basic information of the differential GPS measurement model and their combination to yield refraction corrected range and ionospheric-delay. The improved model of the ionospheric-delay, tightly coupled with range measurement, is described here.

2.1 Differential GPS Measurement

The pseudorange and carrier phase model for both L1 and L2 can be written as (based on [1]):

\[
\tilde{\rho}_1 = r + E_{cm} + \frac{f_2}{f_1} I_a + \eta_1, \quad (1)
\]

\[
\tilde{\rho}_2 = r + E_{cm} + \frac{f_1}{f_2} I_a + \eta_2, \quad (2)
\]

\[
(\tilde{\phi}_1 + N_1)\lambda_1 = r + E_{cm} - \frac{f_2}{f_1} I_a + \beta_1, \quad (3)
\]

\[
(\tilde{\phi}_2 + N_2)\lambda_2 = r + E_{cm} - \frac{f_1}{f_2} I_a + \beta_2 \quad (4)
\]

with \(\rho\) being the pseudorange, \(r\) being the true range from satellite to receiver, \(E_{cm}\) being the common error that can be removed by differential technique, \(f\) being the carrier frequency, \(I_a = \frac{a_0}{f_1}TEC\) being the ionosphere delay, \(\eta\) being the pseudorange noise and multipath error, and \(\beta\) being the carrier phase noise and multipath error. Note here that the receiver clock error is supposed in \(E_{cm}\) for double differential GPS OR a common term for both code and carrier of all satellites.

After differential operation to remove the common error, we can yield following equations:

\[
\Delta \rho_1 = r + \frac{f_2}{f_1} I_a + \eta_1, \quad (5)
\]

\[
\Delta \rho_2 = r + \frac{f_1}{f_2} I_a + \eta_2, \quad (6)
\]

\[
(\Delta \phi_1 + N_1)\lambda_1 = - \frac{f_2}{f_1} I_a + \beta_1, \quad (7)
\]

\[
(\Delta \phi_2 + N_2)\lambda_2 = - \frac{f_1}{f_2} I_a + \beta_2. \quad (8)
\]

Based on eqn. (5) and eqn. (6), we can estimate the ionospheric-corrected pseudorange and ionospheric delay as:

\[
k_1 \Delta \rho_1 - k_2 \Delta \rho_2 = r + k_1 \eta_1 - k_2 \eta_2, \quad (9)
\]

\[
k_c (\Delta \rho_2 - \Delta \rho_1) = I_a + k_c (\eta_2 - \eta_1) \quad (10)
\]

ing ambiguity.” It is also possible to solve for the “floating ambiguity” values as separate states in a least-squares or Kalman filter solution. When the ambiguities are included as states the estimate value can become increasingly accurate as the geometry changes due to satellite motion. This technique also yields an increasingly accurate estimate over time. There are many combinations and variations of the two techniques which can be used to estimate the “floating ambiguity” values. However, all of them involve processing data over a significant time interval. The interval can often be as long as one or two hours before one can be confident that the “floating ambiguity” is accurate enough to yield an accuracy of less than 10 centimeters in the navigated position.

Several differential GPS systems have become available which supply measurements or measurement corrections which can be used in a navigation receiver to obtain navigation results in the 10 centimeter range after the carrier-phase floating ambiguities have been determined to sufficient accuracy. These systems are of several types. The High Accuracy Nationwide Differential GPS System (HA-ND GPS) being developed cooperatively by several U.S. government organizations uses ground based reference sites. This system transmits the corrections to the user using Coast Guard beacons which can reach users at ranges of a few hundred kilometers. John Deere has developed the StarFire™ system which transmits corrections via communication satellites with both a regional wide area correction data stream and a global DGPS correction data stream. The global corrections include an orbital correction stream and, after sufficient time has elapsed, the floating ambiguities can be determined accurately enough can achieve ten-centimeter navigation results.

One of the principal problems of these navigation systems is that anything which causes one of the signals from any of the satellites to be temporarily lost (such as interfering signals, shading or signal blockage, etc.) will cause “cycle slips” in the carrier-phase measurements and the floating ambiguity value will no longer be correct. Without some means of reinitializing the floating ambiguity value a long time interval will be required to determine anew the correct floating ambiguity value. A technique is described herein for reinitializing the floating ambiguity values after brief signal outages such that the long process of determining the new value of the floating ambiguity can be avoided.

The paper presents two L1 backup navigation models in measurement domain and process the performance analysis of both two frequency measurements and one frequency measurements ONLY. The missing measurements at one of the frequencies can be synthesized using the measurements from the retained frequency that was not lost. This is accomplished by modelling the ionospheric refraction effects, which is corrected by the measurements taken while both frequencies are available. When the measurements of one frequency is missing, the divergence between the retained code and carrier phase measurements can be used to detect slowly changing deviations from the ionospheric refraction model.
There are more uncertainties for pseudorange, such as user dynamics and satellite motion;

- the ionospheric-delay is correlated in time and space;

- the pseudorange and the ionospheric-delay are coupled together; when \( \dot{a} \) is well estimated, \( \dot{r} \) can be accurately calculated based on L1 only from eqn. (5) and eqn. (7).

Model the rate of the ionospheric-delay as a first order Gauss-Markov process, which means that the changing rate of the ionospheric-delay is not usually constant, but correlated over short time intervals. The resulting model has:

\[
x_I = \begin{bmatrix} I_a \\ I_a \end{bmatrix},
\]

\[
\dot{x}_I = \begin{bmatrix} 0 & \frac{1}{\tau} \\ 0 & 0 \end{bmatrix} x_I.
\]

## 3 General Model of L1 Backup Navigation

When both L1 and L2 measurements are available, eqn. (9) and eqn. (11) can be used to smooth the ionospheric-corrected pseudorange, and eqn. (10) and eqn. (12) can be used to estimated \( x_I \). This is called decoupled estimation, which is equal to use eqn. (5) to eqn. (8) to estimate following states:

\[
x = \begin{bmatrix} r & \dot{r} & N_1 & N_2 & I_a & I_a \end{bmatrix}^T.
\]

When only L1 measurement is available, for the decoupled estimation, \( N_1 \) can be calculated from \( I_{offset} \) and \( r_{offset} \), then \( N_1 \) and \( I_a \) are used to calculate \( r \). This is equal to use eqn. (5) and eqn. (7) to estimate following states:

\[
x = \begin{bmatrix} r & \dot{r} & N_1 & I_a & I_a \end{bmatrix}^T.
\]

In this section, general models of both the dynamics and the measurements are used to analyze the performance of the states, especially the true range from the satellite to the receiver in the measurement domain, that are used for the navigation solution.

### 3.1 Dynamic and Measurement Models

Building on the results of Section 2, we can define both the dynamic and measurement models and describe their transitions between different modes. Covariance analysis, is performed to evaluate their performance.

#### 3.1.1 Both L1 and L2 Measurements Available

When both L1 and L2 measurements are available, the dynamic model and the measurement model are:

\[
x = \begin{bmatrix} r & \dot{r} & N_1 & N_2 & I_a & I_a \end{bmatrix}^T.
\]
with the same state

When only $L_1$ measurements are available, the measurement equations to initialize the uncertainty of the reduced states. The reduced models are:

$$\begin{bmatrix}
\Delta \rho_1 \\
\Delta \phi_1 \lambda_1 \\
\Delta \phi_2 \lambda_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & \frac{L_2}{f_1} & 0 \\
1 & 0 & -\lambda_1 & 0 & -\frac{L_2}{f_1} & 0 \\
0 & 0 & 0 & 0 & 1 & \frac{1}{\tau_{sa}} \\
0 & 0 & 0 & 0 & 0 & \omega_{I_a}
\end{bmatrix}
x +
\begin{bmatrix}
\eta_1 \\
\eta_2 \\
\beta_1 \\
\beta_2
\end{bmatrix}. \quad (24)
$$

with

$$x = [r \ \dot{r} \ \dot{N}_1 \ \dot{N}_2 \ I_a \ \dot{I}_a]^T. \quad (25)$$

3.1.2 Only L1 Measurements Available

When only $L_1$ measurements are available, the measurement equation becomes

$$\begin{bmatrix}
\Delta \rho_1 \\
\Delta \phi_1 \lambda_1
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & \frac{L_2}{f_1} & 0 \\
1 & 0 & -\lambda_1 & 0 & -\frac{L_2}{f_1} & 0 \\
0 & 0 & 0 & 0 & 1 & \frac{1}{\tau_{sa}} \\
0 & 0 & 0 & 0 & 0 & \omega_{I_a}
\end{bmatrix}
x +
\begin{bmatrix}
\eta_1 \\
\beta_1
\end{bmatrix}. \quad (26)
$$

with the same state $x$ defined above.

Another solution is to reduce $N_2$ from the state and use information of the covariance matrix from the dual frequency measurements to initialize the uncertainty of the reduced states. The reduced models are:

$$\dot{x} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \frac{1}{\tau_{sa}} \\
0 & 0 & 0 & 0 & 0 & \omega_{I_a}
\end{bmatrix}
x +
\begin{bmatrix}
0 \\
\omega_r \\
0 \\
0 \\
0
\end{bmatrix}. \quad (27)
$$

$$\begin{bmatrix}
\Delta \rho_1 \\
\Delta \phi_1 \lambda_1
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & \frac{L_2}{f_1} & 0 \\
1 & 0 & -\lambda_1 & -\frac{L_2}{f_1} & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
x +
\begin{bmatrix}
\eta_1 \\
\beta_1
\end{bmatrix}. \quad (28)
$$

with

$$x = [r \ \dot{r} \ \dot{N}_1 \ I_a \ \dot{I}_a]^T. \quad (29)$$

3.2 Kalman Filter Implementation

The Kalman filter is implemented based on the dynamic and measurement equations listed above. Since the dynamic model is for continuous mode, the discrete transition matrix, $\Phi = e^{Ft}$, needs to be calculated before implementation.

The time update of the Kalman filter is

$$P_{k+1} = \Phi_k P_k \Phi_k^T + Q_{k-1}, \quad (28)$$

$$x_{k+1} = \Phi_k x_k, \quad (29)$$

$$P_{k+1}^+ = (I - K_{k+1} H_{k+1}) P_{k+1}. \quad (30)$$

The measurement update of the Kalman filter is

$$x_{k+1}^* = x_{k+1} - K_{k+1} (Z_{k+1} - H_{k+1} x_{k+1}), \quad (31)$$

$$P_{k+1}^* = |I - K_{k+1} H_{k+1}| P_{k+1}. \quad (32)$$

3.3 Performance Analysis

The performance analysis is based on the covariance analysis of eqn. (28), eqn. (30), and eqn. (32) with $\sigma_\rho = 0.65 m, \sigma_\phi = 0.01 m, \sigma_v = 0.03 m/s, \sigma_{I_a} = 0.001 m/s$, and $\tau_{sa} = 100 s$. The result is shown in Fig. 1. For the first hour from sample 1 to sample 3600, both $L_1$ measurements and $L_2$ measurements are available, the standard deviation of six states are 0.033 m, 0.012 m/s, 0.279 cycle, 0.278 cycle, 0.030 m, and 0.0021 m/s, which are listed in the first row of Table 1. After the first hour, $L_2$ measurements are dropped and only $L_1$ measurements are available, the standard deviation are transferred to the new values 0.219 m, 0.032 m/s, 0.278 cycle, 0.276 cycle, 0.134 m, and 0.0050 m/s, which are listed in the second row of Table 1. Figure 2 shows the transition process of dropping $L_2$ measurements with $L_1$ measurements only. The performance analysis shows:

- when both $L_1$ and $L_2$ measurements are available, the true range and ionospheric delay yield cm accuracy, their rates reach mm accuracy, and both integer ambiguities are within 0.3 cycles;
- when only $L_1$ measurements are available, the true range and ionospheric delay degrade to 20 cm and 13 cm, their rates degrade to 3 and 0.5 mm, and both integer ambiguities do not change.

<table>
<thead>
<tr>
<th>Meas.</th>
<th>$\sigma_r$</th>
<th>$\sigma_r^*$</th>
<th>$\sigma_{N_1}$</th>
<th>$\sigma_{N_1}$</th>
<th>$\sigma_1$</th>
<th>$\sigma_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1L2</td>
<td>0.033</td>
<td>0.017</td>
<td>0.279</td>
<td>0.278</td>
<td>0.030</td>
<td>0.0021</td>
</tr>
<tr>
<td>L1</td>
<td>0.219</td>
<td>0.032</td>
<td>0.278</td>
<td>0.276</td>
<td>0.134</td>
<td>0.0050</td>
</tr>
</tbody>
</table>

Table 1: Performance Result of General Model
4 Simplified Model of L1 Backup Navigation

Since the model of Section 3 is implemented by each satellite, calculating efficiency is important for the GPS receiver, especially embedded within the OEM. Also, for the same available set of the measurements, the less number and the better definition of the states, the better results for state estimation.

In this section, the simplified model of L1 backup navigation is presented. To simplify the implementation, the estimation of \( \hat{r} \) still uses the decoupled method based on Eqns. (13), (15) and (17); while the simplified three states are defined as:

\[
\dot{x} = \begin{bmatrix} N_c & I_a & I_a \end{bmatrix}^T.
\]

with \( N_c = N_1\lambda_1 - N_2\lambda_2 \)

4.1 Both L1 and L2 Measurements Available

When both L1 and L2 measurements are available, the dynamic equation is:

\[
\dot{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 - \frac{1}{\tau_a} \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \eta_2 - \eta_1 \\ \beta_2 - \beta_1 \end{bmatrix},
\]

and the measurement model can be written as:

\[
\begin{bmatrix} \Delta \rho_2 - \Delta \rho_1 \\ \Delta \phi_2\lambda_2 - \Delta \phi_1\lambda_1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{k_2} & 0 \\ 1 - \frac{1}{k_2} & 0 \end{bmatrix} x + \begin{bmatrix} \eta_2 - \eta_1 \\ \beta_2 - \beta_1 \end{bmatrix}
\] (34)

which is based on eqns. (10) and (12). These equations can be used to drive a simple three states Kalman filter to estimate the combination of the L1 and L2 integer ambiguity, ionospheric delay, and the delay rate.

4.2 Only L1 Measurements Available

When only L1 measurements are available, \( N_1 \) can be calculated as:

\[
\tilde{N}_1 = \tilde{r}_{offset} - k_2 N_0 \]

The measurement model of L1 only is

\[
\begin{bmatrix} \Delta \rho_1 - (\Delta \phi_1 + \tilde{N}_1)\lambda_1 \end{bmatrix} = \begin{bmatrix} 0 & 2\frac{I_a}{k_1} & 0 \\ \eta_1 - \beta_1 \end{bmatrix} x
\]

When the L2 signal lost and only the L1 signal is available, the synthesized L2 observables can be calculated as:

\[
\begin{align*}
\Delta \tilde{\phi}_2 & = \frac{I_a}{k_1} + \Delta \rho_1, \\
\Delta \tilde{\phi}_2\lambda_2 & = \Delta \phi_1\lambda_1 + (N_1\lambda_1 - N_2\lambda_2) - \frac{I_a}{k_1}.
\end{align*}
\]

This synthesized L2 measurements can be used to combine with L1 measurements to calculate the reflection corrected range.

4.3 L2 Measurements Return

When L2 measurements return, L2 carrier will have a cycle slip. The slipped cycles \( \delta N_2 \) and the new \( N_1\lambda_1 -
Table 2: Performance Result of Simple Model

<table>
<thead>
<tr>
<th>Meas.</th>
<th>$\sigma_{N_1}$ cycle</th>
<th>$\sigma_I$ m</th>
<th>$\sigma_I$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1L2</td>
<td>0.017</td>
<td>0.028</td>
<td>0.0012</td>
</tr>
<tr>
<td>L1</td>
<td>0.016</td>
<td>0.146</td>
<td>0.0068</td>
</tr>
</tbody>
</table>

$N_2\lambda_2$ can be re-calculated as:

$$\delta N_2 = \text{round}(\Delta\phi_2 + \Delta\hat{\phi}_2), \quad (39)$$

$$(N_1\lambda_1 - N_2\lambda_2) = (N_1\lambda_1 - N_2\lambda_2)_{old} + \delta N_2\lambda_2. \quad (40)$$

with $(N_1\lambda_1 - N_2\lambda_2)_{old}$ being the state of $N_c$ before L2 measurements lost. This process, described herein, is called the floating ambiguity reinitialization after signal outages. This avoids the long process of determining the new value of the floating ambiguity via smoothing.

4.4 Performance Analysis

The performance analysis is based on the covariance analysis of eqn. (28), eqn. (30), and eqn. (32) with $\sigma_p = 0.92\text{m}$, $\sigma_\phi = 0.014\text{m}$, $\sigma_{\dot{\phi}} = 0.0012\text{m/s}$, and $\tau_{\dot{\phi}} = 100\text{s}$. The result is shown in Fig. 3. For the first hour from sample 1 to sample 3600, both L1 measurement and L2 measurements are available, the standard deviation of three states are $0.017 \text{cycle}$, $0.028 \text{m}$, $0.0012 \text{m/s}$, as listed in the first row of Table 2. After the first hour, L2 measurements are dropped and only L1 measurements are available, the standard deviation are transited to the new values as $0.016 \text{cycle}$, $0.146 \text{m}$, $0.0068 \text{m/s}$, which are listed in the second row of Table 2. The performance analysis shows:

- when both L1 and L2 measurements are available, the ionospheric delay yields cm accuracy, its rate reaches $\frac{mm}{s}$ accuracy, and both integer ambiguities are within 0.02 cycles;
- when only L1 measurements are available, the ionospheric delay degrades to 15 cm, its rate degrades to $\frac{mm}{s}$, and the combined integer ambiguity does not change.

5 Conclusion

To achieve high accuracy ($\leq 10\text{cm}$) differential GPS with long baseline (\geq 100\text{Km}), such as StarFire\textsuperscript{TM}, both L1 and L2 measurements are required to form ionosphere-corrected range measurements to estimate the floating ambiguity due to the ionospheric divergence between code and carrier. The process usually take more than 0.5 hour to yield the good result via the phase smoothing code. L2 measurements are easily lost due to the less signal strength and codeless/semicodeless decoding.

Figure 3: Performance analysis of Simple L1 backup

General models and simplified models in the measurement domain are proposed for both L1 and L2 measurements and L1 only measurements. Performance analysis is performed for both models which show that the simplified model can yield similar performance with three states fewer. The simplified models with three states provide computational efficiency for the GPS OEM target.

When L2 measurements are lost, they can easily be synthesized based on the L1 measurements, the estimated ionospheric delay, and the combination of integer ambiguity. When L2 measurements return, the slipped cycles on L2 can be detected based on the new measurements and the estimated states.

The synthesized measurements can be either of the L1, L2 or L5, if any one of the three frequencies is retained.

References

