GPS Multipath Mitigation in Measurement Domain and
Its Applications for High Accuracy Navigation

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Biography

Dr. Yunchun Yang is a senior systems engineer at NavCom Technology Inc. He received B.S. and M.S. degrees from Harbin Engineering University, China in 1993 and 1997, and the Ph.D. degree from University of California, Riverside in 2001. His research interest are: advanced GPS signal processing, DGPS RTK, attitude determination, and tight GPS INS integration.

Mr. Ron Hatch is the Director of Navigation Systems at NavCom Technology, a company of which he was a co-founder. He has developed a number of innovative techniques for processing GPS measurements and has obtained more than a dozen patents related to GPS. Ron is a past president of the Institute of Navigation, is a Fellow of the ION and has received the Kepler and Thurlow awards from the ION.

Mr. Tenny Sharpe is the Director of Advanced Development at NavCom Technology Inc. Mr. Sharpe received a B.S. in Physics from Case Institute of Technology and a M.S. in Computer Science from the University of California, Los Angeles. Mr. Sharpe has over 30 years experience in the development of aerospace and industrial electronics. His specializations are software and systems design for GPS navigation systems.

Abstract

GPS receiver tracks both the satellite code and the carrier signals and decodes them into range and delta range from the satellite to the receiver. There are various sources of errors corrupting the code and carrier measurements. They are mainly: ionosphere delay, clock error, ephemeris error, troposphere delay, receiver noise, and multipath. Most of these errors can be removed by differential techniques due to their space correlation. The ionosphere delay error can also be estimated from the measurements on different frequencies. However, the receiver noise and multipath error are independent for each receiver. Since the receiver noise is white-like and can be smoothed via low pass filter, the multipath error is the main source of the error for high accuracy navigation. This is especially true for the code measurement, where the error could have a magnitude of tens of meters.

This paper presents a technique for GPS multipath mitigation in the measurement domain and its application for high accuracy navigation, such as the WADGPS Starfire developed by NavCom. The technique is based on the time-correlation properties of the GPS multipath error. Two alternatives, a four-state Kalman filter and a simple two-state Kalman filter, are used for each GPS satellite. These are: GPS range, delta range, multipath error, and ambiguity. GPS code measurement and carrier phase measurement are used to drive the filter. Special techniques are used for the state initialization and relationships among different satellites. Experimental results are presented showing that the technique can not only improve the performances, such as the accuracy and pull-in time of phase based WADGPS, for stationary applications but also enhance the performances for dynamic applications.

1 Introduction

The GPS receiver generates a specific pseudorandom code signal (PRN) at the defined frequency, which is used to correlate with the received satellite signal. The locally generated signal is processed with a frequency search (or FFT) to remove the doppler, with a code shifting to achieve maximum correlation, an AFC loop to obtain frequency lock, and a COSTAS loop for the phase lock. When all these processes have reached the steady state, the channel is locked. To keep the channel locked the satellite signal, these processes calculate the tracking errors and feedback the code, frequency, and phase correction to maintain the maximum correlation between the satellite signal and the locally generated signal. The locally generated signal, which is known, is used to decode code range and delta range of carrier phase.

Multipath errors are due to the reflected signals from surfaces along the direct signal path from the satellite to the receiver. The reflected signals shift the correlation peak and corrupt the theoretically symmetric receiver correlation envelope. Both of these changes to the correlation envelope result in erroneous pseudorange and phase measurements. As stated in [2], the multipath
errors affect both stationary and mobile receivers. For mobile receivers, the signal path and reflecting geometries are changing, so the correlation time of multipath errors for roving receivers is significantly smaller than for stationary receivers. In both stationary and mobile applications, the unknown characteristics of the direct and reflected signal paths make modelling (and prediction) of the multipath errors an infeasible task. Code multipath can result in pseudo-range errors of 0.1-3.0 m depending on various design and antenna siting factors, while the carrier phase multipath error is usually less than a few centimeters.

Several methods of multipath estimation and mitigation have been investigated and implemented. Existing methods can be classified into three broad categories according to the signals that are processed: the first category is based on the radio frequency signals; the second category is based on the available baseband signals; the third category is based on final measurements (code, carrier phase) after receiver baseband processing. The first category includes, but is not limited to, using a choke-ring GPS antenna [12] and using multiple GPS antennae for multipath mitigation[10]. The second category includes, but is not limited to, narrow correlator technology [13, 5], double delta correlator [7, 5], early/late slope technique (ELS) [11, 5], early1/early2 (E1/E2) tracking [14, 5]. The third category can be used in most scenarios without requiring access to the baseband and Radio Frequency signal inside the GPS receiver. This category includes, but is not limited to, processing the previous day measurements as correction for the next day’s measurements, analyzing the Signal to Noise Ratio (SNR) of the GPS measurements [1], estimating the multipath error by proper modeling[2, 8], and using multiple GPS antennae for multipath mitigation[10].

This paper presents a technique for GPS multipath mitigation in the measurement domain which belongs to the third category with the motivation for high accuracy navigation using Wide-Area Differential GPS (WADGPS), such as the Starfire system developed by NavCom. The technique is based on the time-correlation properties of the GPS multipath error [2] for real-time applications. The technique can be implemented for GPS receivers from different manufactures. To calculate efficiently for real-time implementation, two or four state Kalman filter is used for each GPS satellite, instead of eight-state used in [2]. Also, to account for the different correlation time and magnitude of the multipath error, special adaptive and initialization techniques are used as detailed in Section 3.3 and Section 3.4.

2 Background

The section describes the model of the multipath, the model of the differential GPS measurements, and the measurement combination to yield ionosphere-free range and multipath error.

2.1 Model of Multipath and its Properties

As defined in Chapter 14 of [9], the direct GPS signal can be written as:

\[ s_d(t) = AP(t) \sin \omega_0 t \] (1)

with \( A \) being the amplitude of the GPS signal, \( P(t) \) being the \( P \) modulated code, \( t \) being the time delay, and \( \omega_0 \) being phase rate. All the reflected GPS signal can be written as:

\[ s_m(t) = \sum_{k=0}^{n} \alpha_k AP(t - \delta_k) \sin(\omega_0 t + \theta_k) \]

\[ = \alpha_m AP(t - \delta_m) \sin(\omega_0 t + \theta_m) \] (2)

with \( \alpha_m \) being the combined multipath relative amplitude, \( \delta_m \) being the combined multipath relative time delay, and \( \theta_m \) being the combined multipath relative phase.

Therefore, the true received GPS signal at the receiver is:

\[ s(t) = s_d(t) + s_m(t) \]

\[ = AP(t) \sin \omega_0 t + \alpha_m AP(t - \delta_m) \sin(\omega_0 t + \theta_m). \] (3)

From eqn. (3), there is a time-correlation between \( s_d(t) \) and \( s_m(t) \) correlation determined by \( \alpha_m, \delta_m, \text{ and } \theta_m \) when the GPS receiver performs the signal correlation processes.

2.2 Combing L1 and L2 Measurements

After differential calculation, the L1 and L2 pseudorange and carrier phase can be written as:

\[ \nabla \rho_1 = r + \frac{f_2}{f_1} I_n + MP_1 + \eta_1 \] (4)

\[ \nabla \rho_2 = r + \frac{f_1}{f_2} I_n + MP_2 + \eta_2 \] (5)

\[ (\nabla \phi_1 + N_1) \lambda_1 = r - \frac{f_2}{f_1} I_n + n_1 \] (6)

\[ (\nabla \phi_2 + N_2) \lambda_2 = r - \frac{f_1}{f_2} I_n + n_2 \] (7)

where \( \nabla \rho \) is differential code measurement, \( \nabla \phi \) is differential phase measurement, \( r \) the true range from GPS receiver to satellite, \( N \) is the integer ambiguity, \( f \) is carrier phase frequency, \( I_n \) is the ionosphere delay error,
MP is the code multipath error, \( \eta \) is the code noise, \( n \) is the phase noise plus phase multipath which is ignore herein, subscript 1 is for GPS \( L_1 \) signal, and subscript 2 is for GPS \( L_2 \) signal.

Combining eqn. (4) to eqn. (5) to remove the ionospheric term of \( I_a \) and re-arranging yields:

\[
\frac{f_1}{f_2} \nabla \rho_1 - \frac{f_2}{f_1} \nabla \rho_2 = \left( \frac{f_1}{f_2} - \frac{f_2}{f_1} \right) r + \left( \frac{f_1}{f_2} MP_1 - \frac{f_2}{f_1} MP_2 \right) + \left( \frac{f_1}{f_2} \eta_1 - \frac{f_2}{f_1} \eta_2 \right),
\]

which can be written as:

\[
f_\frac{1}{2} \nabla \rho_1 - f_\frac{2}{2} \nabla \rho_2 = f_\frac{1}{H} r + MP + \eta \tag{9}
\]

with \( f_\frac{1}{2} = \frac{f_1}{f_2} \), \( f_\frac{1}{2} = \frac{f_2}{f_1} \), \( f_2 = \frac{f_2}{f_1} - \frac{f_2}{f_1} MP_1 - \frac{f_2}{f_1} MP_2 \), and \( \eta = \left( \frac{f_1}{f_2} \eta_1 - \frac{f_2}{f_1} \eta_2 \right) \).

Combining eqn. (6) to eqn. (7) to remove the term of \( I_a \) and re-arranging yields:

\[
\nabla \phi_1 \lambda_2 - \nabla \phi_2 \lambda_1 = \left( \frac{f_1}{f_2} - \frac{f_2}{f_1} \right) r - (N_1 \lambda_2 - N_2 \lambda_1) + \left( \frac{f_1}{f_2} n_1 - \frac{f_2}{f_1} n_2 \right),
\]

which can be re-written as:

\[
\nabla \phi_1 \lambda_2 - \nabla \phi_2 \lambda_1 = f_\frac{1}{H} r - N + n \tag{11}
\]

with \( N = (N_1 \lambda_2 - N_2 \lambda_1) \) and \( n = \left( \frac{f_1}{f_2} n_1 - \frac{f_2}{f_1} n_2 \right) \).

Eqn. (9) and eqn. (11) provide the basic measurement equations for multipath estimation. Combining eqn. (9) and eqn. (11) to remove the range term of \( r \), yields:

\[
(f_\frac{1}{2} \nabla \rho_1 - f_\frac{1}{2} \nabla \rho_2) - (\nabla \phi_1 \lambda_2 - \nabla \phi_2 \lambda_1) = MP + N + w \tag{12}
\]

with \( w = \eta + n \).

3 Methodology

This section details the methodology of multipath estimation.

3.1 General Model of Multipath

Model the multipath error as a first order Gauss-Markov process as presented in [2], which means that the multipath error is not usually constant, but correlated over some time intervals. The resulting model has:

\[
\dot{x}_m = -\frac{1}{\tau_m} x_m. \tag{13}
\]

Use of both eqn. (9) and eqn. (11) to drive the filter will therefore require the state to be defined as \( x = [r, \dot{r}, MP, N]^T \) for low dynamic. For high dynamic, the state can be defined as \( x = [r, \dot{r}, MP, N]^T \). Without loss the generality, use \( x = [r, \dot{r}, MP, N]^T \) to evaluate the multipath estimation. The dynamic model can be presented as:

\[
x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{\tau_v} & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_m} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x. \tag{14}
\]

where \( \tau_v \) is the time correlation of velocity and \( \tau_m \) is the time correlation of multipath error with \( \tau_m = 60 \sim 960 \)s for evaluation herein. The measurement equation of eqn. (9) can be written as:

\[
f_\frac{1}{2} \nabla \rho_1 - f_\frac{1}{2} \nabla \rho_2 = H_1 x + \eta \tag{15}
\]

with

\[
H_1 = \begin{bmatrix} f_\frac{1}{H} & 0 & 1 & 0 \end{bmatrix}.
\]

The measurement equation of eqn. (11) can be written as:

\[
\nabla \phi_1 \lambda_2 - \nabla \phi_2 \lambda_1 = H_2 x + n \tag{17}
\]

with

\[
H_2 = \begin{bmatrix} f_\frac{1}{H} & 0 & 0 & -1 \end{bmatrix}.
\]

As we know the worst scenario for multipath error is the system in stationary mode with reflected signals in addition to the direct signal. Supposing that the system is stationary with \( \frac{1}{\tau_v} \) being 0, how the multipath correlation time, \( \tau_m \), and the multipath processing noise, \( \sigma_m \), affect the performance of state estimation with \( \sigma_{cd} = 1m \), \( \sigma_{phs} = 0.01m \), and \( \sigma_v = 0.01m/s \) is evaluated herein.

Figure 1 shows that all states converges with different duration for different correction time, \( \tau_m \). The range rate converges quickly due to the accurate phase measurement. The shorter the correlation time, the faster the convergence. The uncertainty of the range, \( r \), and the ambiguity, \( N \), are strongly dependent on the multipath correction time, \( \tau_m \). Though the uncertainty of the multipath via its correlation time is not as sensitive as the range and ambiguity values. It is important for range estimation based on eqn.(11) and the ambiguity, \( N \).

Figure 2 shows that all states are observable with different uncertainties. The range rate converges quickly due to the accurate phase measurement. The smaller the magnitude of the multipath error, the more accurately the states are determined.

3.2 Simple Model of Multipath

For real-time implementation, computational efficiency is important. Use of eqn. (12) to drive the filter will
The dynamic model can be presented as:

\[ \dot{x} = \begin{bmatrix} -\frac{1}{\tau_m} & 0 \\ 0 & 0 \end{bmatrix} x. \]  

(19)

The measurement equation of eqn. (12) can be written as:

\[ (f_1 \nabla \rho_1 - f_2 \nabla \rho_2 - (\nabla \phi_1 \lambda_2 - \nabla \phi_2 \lambda_1) = Hx + w \]  

(20)

with

\[ H = \begin{bmatrix} 1 & 1 \end{bmatrix}. \]  

(21)

Similar to Section 3.1, we want to evaluate how the multipath correlation time, \( \tau_m \), and the multipath processing noise, \( \sigma_m \), affect the performance of state estimation with \( \sigma_{cd} = 1 \text{m} \) and \( \sigma_{phs} = 0.01 \text{m} \).

Figure 3 shows that both multipath and ambiguity converge with different duration for different correlation time, \( \tau_m \). The shorter the correlation time, the faster the convergence. The uncertainty of the ambiguity, \( N \), are strongly dependent upon the multipath correlation time, \( \tau_m \), which is important for range estimation based on eqn.(11) and the ambiguity, \( N \).

3.3 Adaptable Time Correlation Calculation

Figure 1 and Figure 3 show that the time correlation of multipath, \( \tau_m \), does affect the state estimation time and accuracy. The correlation time can be determined by an experimental equation as:

\[ \tau_m = \tau_m_0 + \alpha \left| \frac{\partial p}{\partial t} \right| \]

(22)

with \( \tau_m_0 \) being the basic multipath correlation time, \( \alpha \) being the constant scalar, \( \tau_p \) being the post residual, \( t \) being the time, \( \partial \) being the partial differential, and \( \partial x \) being the smoothed partial differential of \( x \).

Figure 2 shows that all states are observable with different uncertainties. The smaller the magnitude of the multipath error, the more accurate the state estimation.

3.4 Multipath Magnitude of Processing Noise

Figure 2 and Figure 4 show that the magnitude of multipath, \( \sigma_m \), does affect the state convergence and accur-
Figure 3: Performance analysis of each state with different multipath correlation time and the same magnitude $\sigma_m = 0.5m$ for the simple model. From bottom: first—($\tau_m = 60s$), second—($\tau_m = 120s$), third—($\tau_m = 240s$), forth—($\tau_m = 480s$), and fifth—($\tau_m = 960s$).

Figure 4: Performance analysis of each state with the same multipath correlation time, $\tau_m = 240s$, and different magnitude for the simple model. From bottom: first—($\sigma_m = 0.1m/\sqrt{Hz}$), second—($\sigma_m = 0.5m/\sqrt{Hz}$), third—($\sigma_m = 1.0m/\sqrt{Hz}$), forth—($\sigma_m = 2.0m/\sqrt{Hz}$), and fifth—($\sigma_m = 3.0m/\sqrt{Hz}$).

Table 1: Performance Result of Multipath Filter

<table>
<thead>
<tr>
<th>prn 2</th>
<th>prn 3</th>
<th>prn 5</th>
<th>prn 6</th>
<th>prn 7</th>
<th>prn 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-9.61</td>
<td>-6.22</td>
<td>-4.41</td>
<td>-7.39</td>
<td>-3.75</td>
</tr>
<tr>
<td>STD</td>
<td>0.011</td>
<td>0.023</td>
<td>0.010</td>
<td>0.022</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Table 2: Performance Result of Exponential Filter

<table>
<thead>
<tr>
<th>prn 2</th>
<th>prn 3</th>
<th>prn 5</th>
<th>prn 6</th>
<th>prn 7</th>
<th>prn 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-9.33</td>
<td>-6.61</td>
<td>-4.54</td>
<td>-7.65</td>
<td>-3.37</td>
</tr>
<tr>
<td>STD</td>
<td>0.063</td>
<td>0.111</td>
<td>0.029</td>
<td>0.050</td>
<td>0.060</td>
</tr>
</tbody>
</table>

The multipath magnitude can be determined by an equation as:

$$\sigma_m = \sigma_{m_0} + \beta \bar{\sigma}_r$$  \hspace{1cm} (23)

with $\sigma_{m_0}$ being the basic multipath magnitude standard deviation, $\beta$ being the constant scalar, $\sigma_r$ being the standard deviation of the post residual, and $\bar{x}$ being the smoothed $x$.

4 Experimental Results

24 hours data was collected from the StarFire developed by NavCom and the measurement is formed based on eqn. (12). The simple, two-state, multipath model is used to process the multipath state, $MP$, and ambiguity, $N$ with $\tau_m = 600s$ and $\sigma_m = 1m$. To compare with this two-state multipath filter, a traditional exponential filter with $T = 3600s$ is also used to smooth the data.

Figure 5, Figure 6, and Figure 7 show the plots of the results for PRN 2, PRN 5, and PRN 8. These figures show that the simple multipath filter does pull in faster than the exponential filter.

Table 1 and Table 2 show the results of the smoothed ambiguity with multipath filter and exponential filter, respectively. Comparing the simple multipath and exponential filter, the simple multipath filter yields a smaller standard deviation than the exponential filter. However, there is bias between the mean value of the simple multipath and exponential filter. This bias will be compensated by the phase bias state of the StarFire system software.

5 Conclusion

This paper presents a technique for GPS multipath mitigation in measurement domain and its applications for Wide Area Differential GPS, such as the Starfire develop-
Figure 5: State estimates for PRN2. Top–multipath estimate of the simple multipath model, Bottom–ambiguity estimate of the simple multipath model with solid line and ambiguity estimate of the exponential filter with solid-dot line

Figure 6: State estimates for PRN5. Top–multipath estimate of the simple multipath model, Bottom–ambiguity estimate of the simple multipath model with solid line and ambiguity estimate of the exponential filter with solid-dot line

ded by NavCom. The technique is based on the time-correlation properties of the GPS multipath error. A four-state Kalman filter and a simple two-state Kalman filter are derived and analyzed. Performance is evaluated based on the multipath correlation time, \( \tau_m \), and multipath process noise magnitude, \( \sigma_m \). Experimental equations for real-time implementation of multipath correlation time, \( \tau_m \), and multipath process noise magnitude, \( \sigma_m \) are given. Experimental results are presented showing that the technique can not only improve the accuracy performance but also the pull-in time of the phase based WADGPS.

References


Figure 7: State estimates for PRN8. Top—multipath estimate of the simple multipath model, Bottom—ambiguity estimate of the simple multipath model with solid line and ambiguity estimate of the exponential filter with solid-dot line


